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B. Sc. Part II Paper - IV
Physics Hons
Current electricity

LR Circuit

Growth and Decay of Current in LR Circuit :-

Consider a circuit containing a coil of inductance L, a non-inductive resistor of resistance R connected in series along with a battery of emf E and a two way key Sab. The battery E may be connected or removed from the circuit by the help of switch S.

Growth of Current :

The battery E is included in the circuit by throwing the switch S to a, so that the circuit aSLREa is completed.

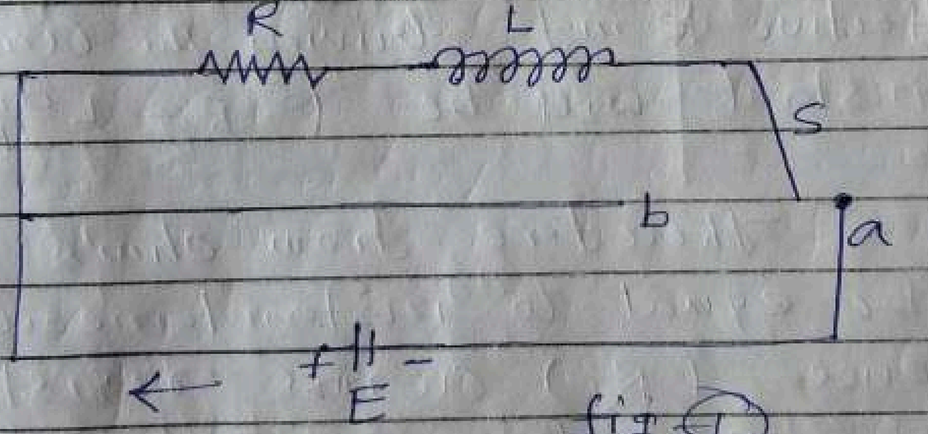

fig (1)

As soon as the battery is included in the circuit, the current begins to flow and a magnetic flux is linked with the coil. During the variable state, when the current is growing in the circuit, The flux linked with the coil changes and so an e.m.f. is induced in the coil which opposes the rise of current in the circuit. Hence the current does not attain its final steady value i_0 instantaneously, but grows at the rate depending upon the inductance and resistance of the circuit.

During the variable state of growth of current, let i be the instantaneous current and di/dt, the rate of rise of current after time t. The emf induced in the coil is $L\, di/dt$. This opposes the applied e.m.f. Hence the effective e.m.f. driving the circuit at this instant will be $\left(E - L\frac{di}{dt}\right)$.

Therefore, from ohm's law, this must be equal to instantaneous potential difference (Ri) across resistance.

Hence $$E - L\frac{di}{dt} = Ri \quad \text{—— (1)}$$

or $$E - Ri = L\frac{di}{dt}$$

or $$dt = \frac{L\,di}{E - Ri}$$

integrating we get

$$t = \frac{L \log_e (E - Ri)}{-R} + A \quad \text{—— (2)}$$

A being constant of integration

i.e. $t = -\frac{L}{R} \log_e (E - Ri) + A$ — (3)

Using initial condition that at $t = 0$, $i = 0$, equation (3) gives

$0 = -L/R \log_e E + A$

i.e. $A = \frac{L}{R} \log_e E$

Substituting this values of A in (3), we get

$t = -\frac{L}{R} \log_e (E - Ri) + \frac{L}{R} \log_e E$

$= -\frac{L}{R} \left[\log_e \left(\frac{E - Ri}{E} \right) \right]$

i.e. $\log_e \left(\frac{E - Ri}{E} \right) = -\frac{R}{L} t$

i.e. $\frac{E - Ri}{E} = e^{-(R/L) t}$

This gives $i = \frac{E}{R} \left[1 - e^{-\left(\frac{R}{L}\right) t} \right]$

$i = i_0 \left[1 - e^{-(R/L) t} \right]$ — (4)

where $i_0 = E/R$ is the final steady value of current in the circuit. equn (4) represents the equation of rise of current in the LR circuit.

The quantity L/R has the dimension of time and is known as inductive time constant of the circuit.

After time $t = L/R$, the current in circuits is

$$i = i_0 (1 - e^{-1}) = 0.638\, i_0$$

That is the (inductive) time constant of LR circuit may be defined as the time in which current in circuit attains 0.638 (or 63.8%) of its final steady value.

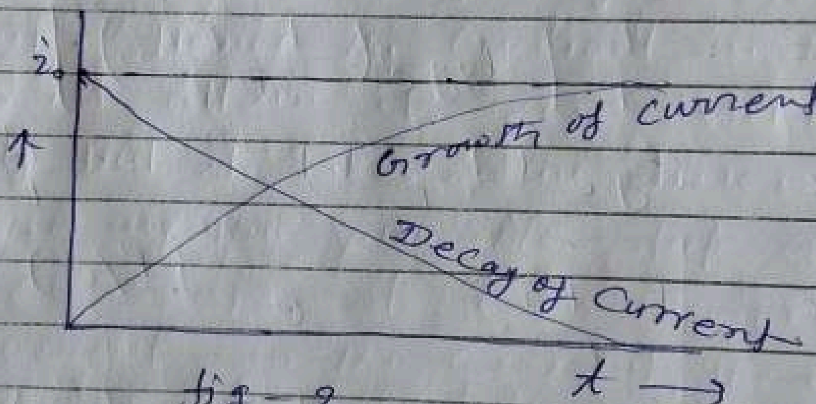

fig-2

From equ^n (4) it is obvious that the current in the circuit rises exponentially. The graph representing the growth of current (i) with time (t) is shown in fig (2).

The rate of growth of current in the circuit is given by

VKSU

$$\frac{di}{dt} = \frac{d}{dt}\left[i_0 \left(1 - e^{-(R/L)t}\right)\right]$$

$$= i_0 \frac{R}{L} e^{-\left(\frac{R}{L}\right)t}$$

$$= R/L\,(i_0 - i) \quad [\text{Using } (4)] \rightarrow (5)$$

From this equⁿ it is clear that greater is the ratio (R/L) or smaller is the ratio (L/R) greater is the rate of growth of current in the circuit Hence for smaller inductor time constant (L/R) the current in the circuit attains its maximum steady value more rapidly.